Diagram Understanding in Geometry Questions

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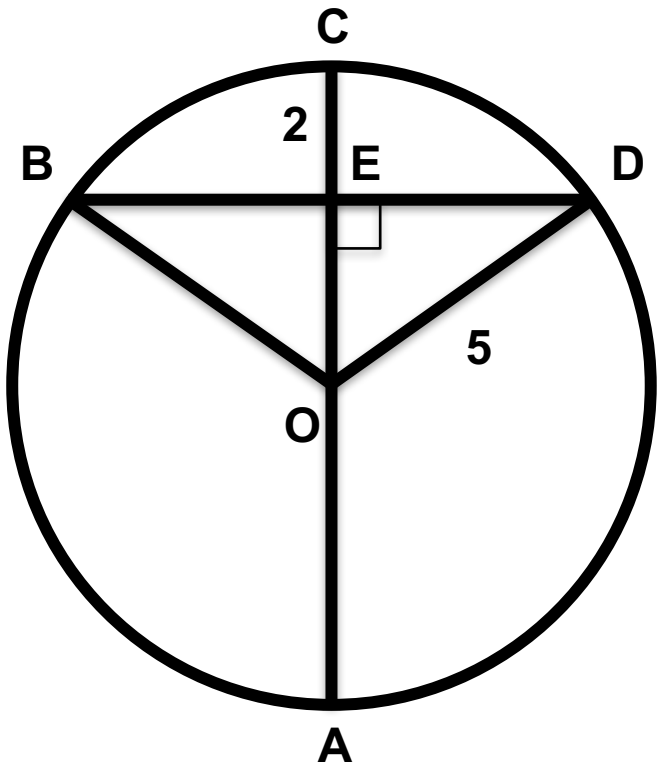
Sample Geometry Question



Sample Geometry Question

In the diagram at the right, circle O has a radius of 5, and CE = 2. Diameter AC is perpendicular to chord BD. What is the length of BD?

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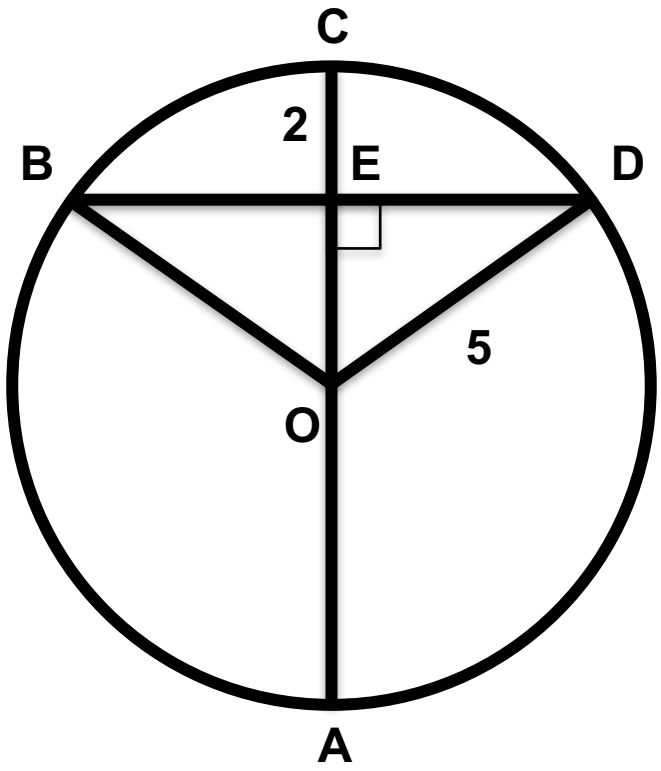

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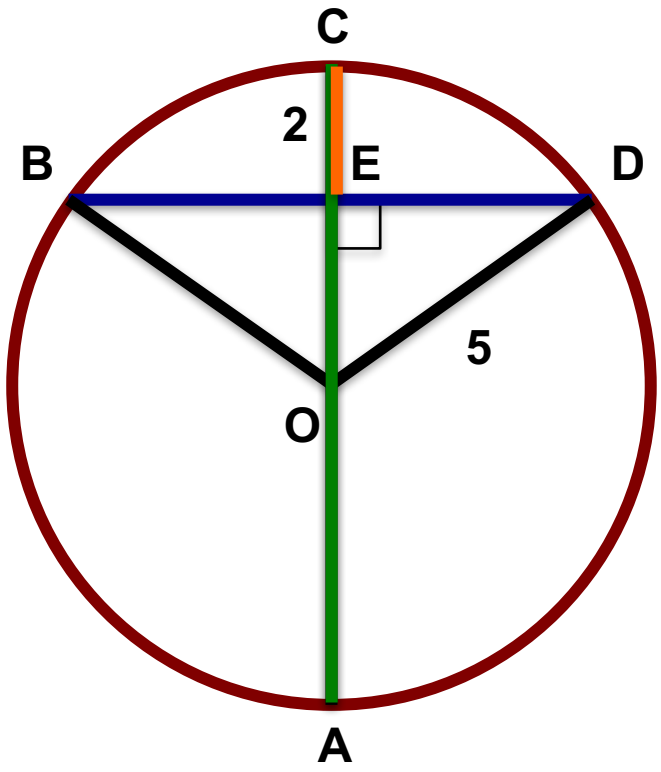

Diagram Understanding

1. Discovering locations of visual elements.
2. Discovering their geometric properties.
3. Aligning them with the text.

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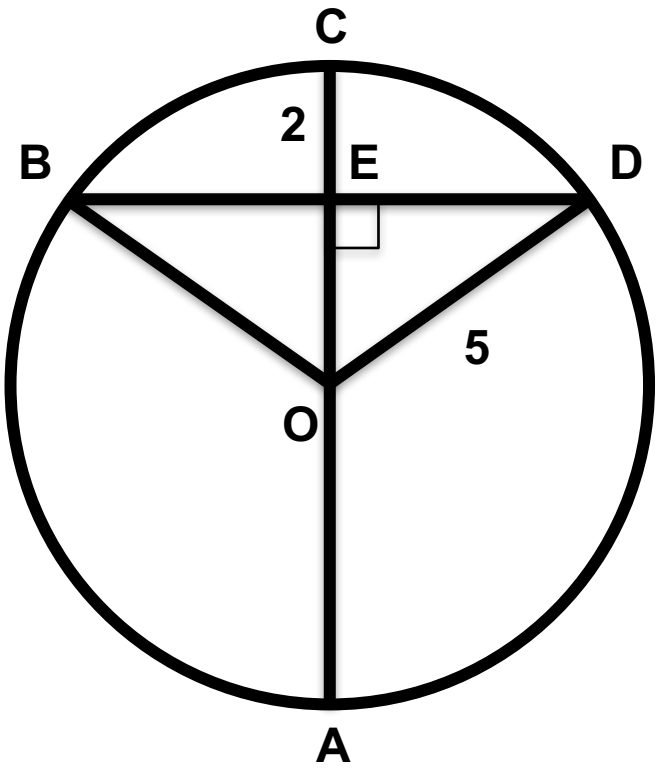

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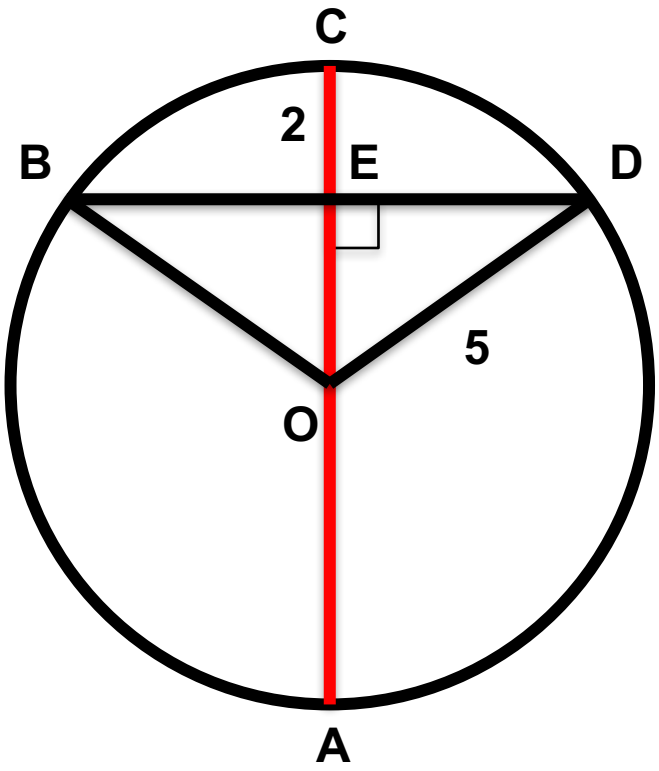
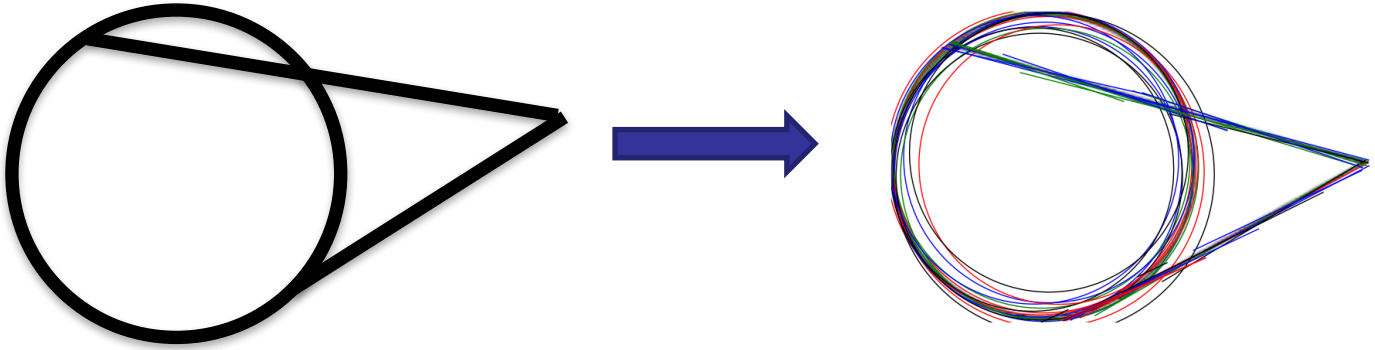


Standard Vision Techniques

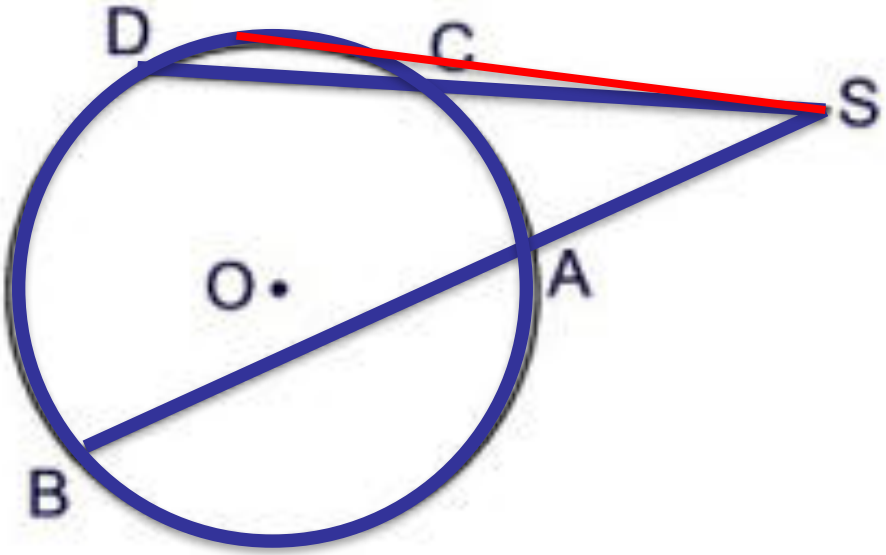
- Usually uses **Hough Transform**:
  - **Detection**: Finds hundreds of lines and circles, each with some confidence level.



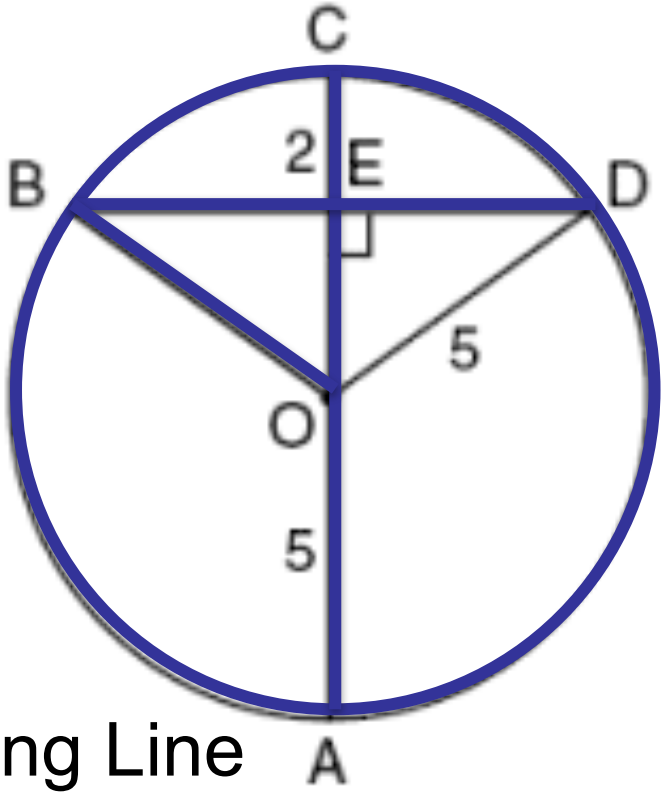
  - **Filtering**: Removes those with low confidence (thresholding); removes similar lines and circles (non-maximum suppression).

Problem: Parameter-sensitive (5 params)

Performance of Pure Hough



1 Extra Line



1 Missing Line

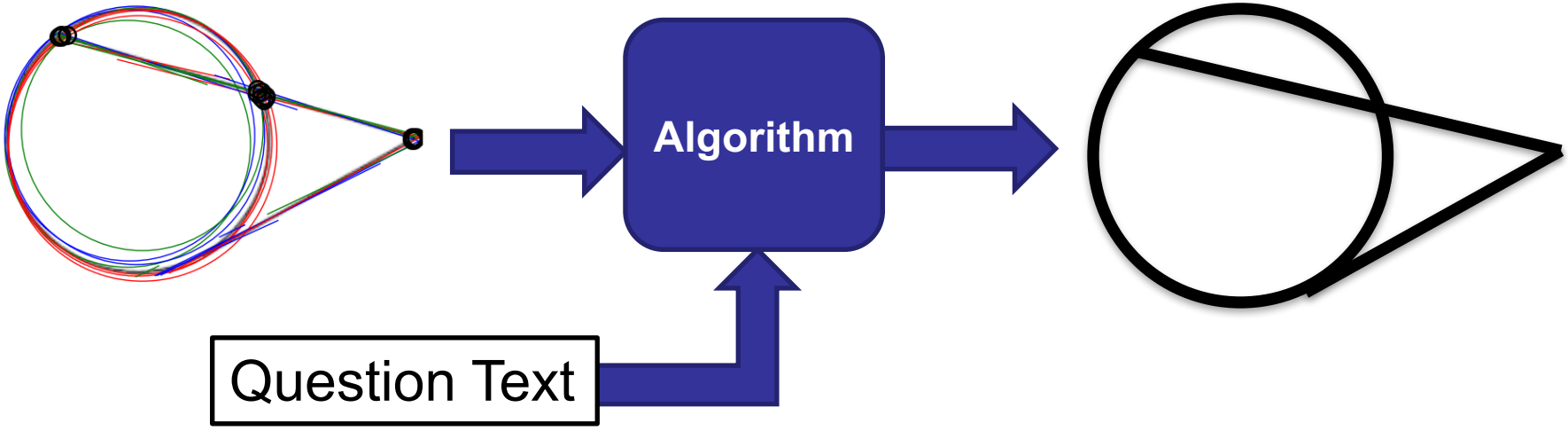
*Same parameters for both figures

Use both **diagram** and **text**

Intuition

- Start with unfiltered lines and circles (primitives, L).

- **Goal**: Find a subset of primitives ($\hat{L}$) that best represents the diagram, using information from ***both text and diagram***.



Evaluation Function

How do we know if $\hat{L}$ represents diagram "*well*"?

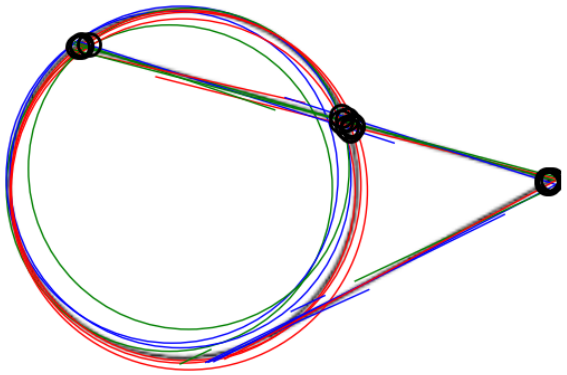
$$F(\hat{L})$$

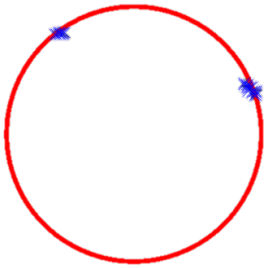
- Pixel **coverage** of primitives P
- Visual **coherence** between primitives C
- **Alignment** with textual information S

Coverage

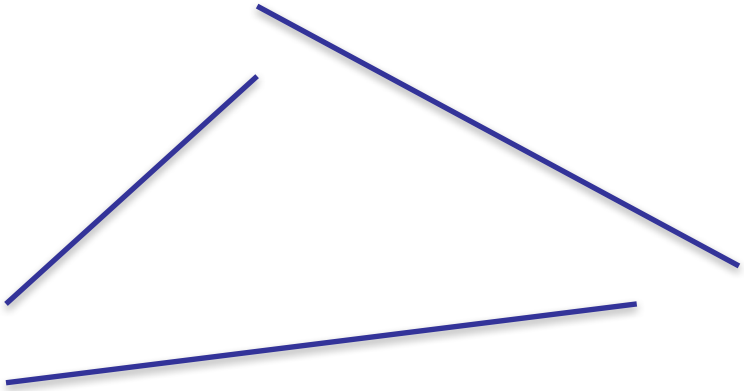
Optimal subset of primitives $\hat{L}$ should explain most of the non-white pixels in the diagram.

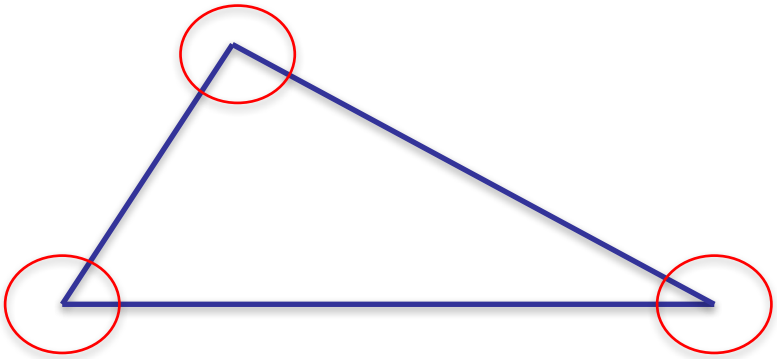
Maximize $P(D,\hat{L}) = \frac{|D_{\hat{L}}|}{|D|} =$ $\frac{\text{\# pixels covered by the set L}}{\text{\# all pixels in the diagram}}$



P(D,  **) > P(D,**  **)**

Visual Coherence





Detected visual elements are visually coherent if they agree on corners.

$$\mathrm{C}(H,\hat{L}) = \frac{\left|H_{\hat{L}}\right|}{\left|H\right|} =$$

$$\frac{\text{\# corners covered covered by L}}{\text{\# all Corners in the diagram}}$$

Alignment

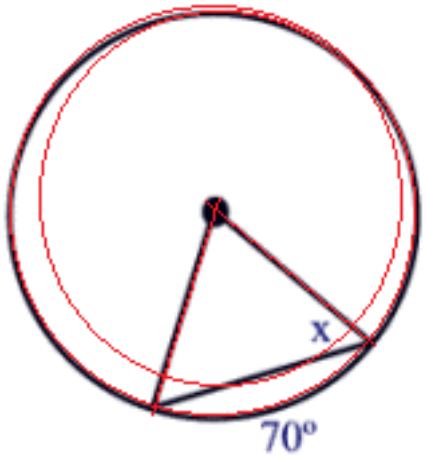
Maximizes alignment between primitives and textual mentions

- Mention `triangle ABC` → three lines `AB`, `AC`, `BC`
- Aligned mention: corresponding primitives are detected

$$\text{Alignment} = \frac{\#\text{ aligned mentions in L}}{\#\text{ all mentions}}$$

- Also penalize redundancy

Text alignment $S(T, \hat{L}) = \frac{|T_{\hat{L}}|}{|T|} - r_{\hat{L}}$



Optimization

$$\hat{L}^* = \underset{\hat{L} \in 2^L}{\operatorname{argmax}} F(\hat{L}, D, H, T) =$$

$$P(D, \hat{L}) \quad \text{Coverage}$$

$$+ C(H, \hat{L}) \quad \text{Visual Coherence}$$

$$+ S(T, \hat{L}) \quad \text{Text alignment}$$

Problem: Optimization requires $2^{|L|}$ operations

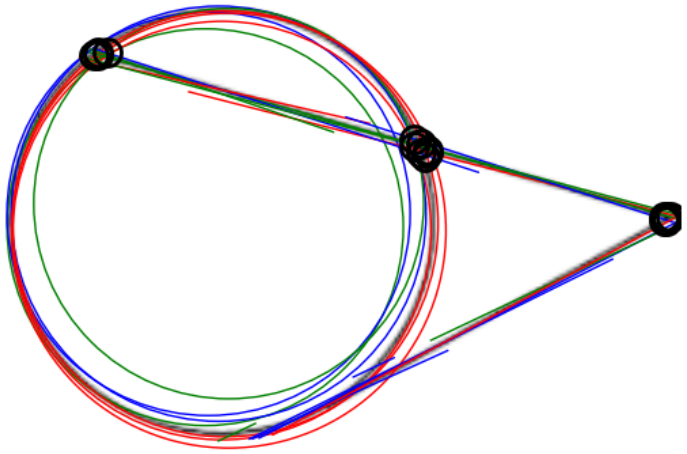
Solution: F is submodular

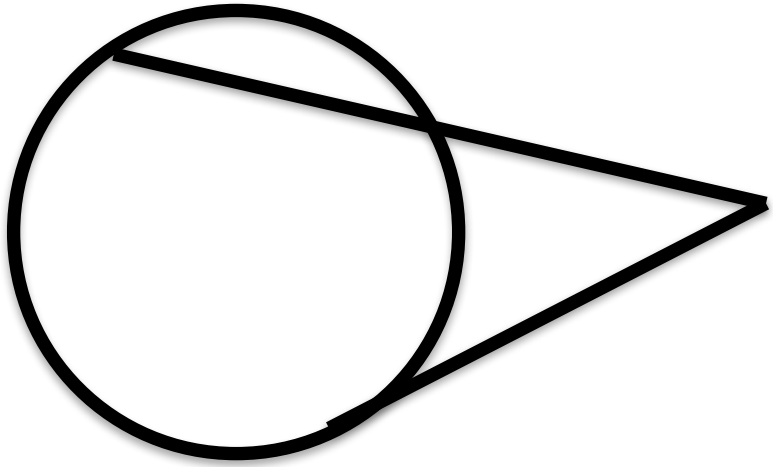
Suboptimal Efficient Algorithm

Greedy Algorithm

- $\geq 1 - \frac{1}{e}$ of maxima, by submodularity of F
- **O(n)**, where n is the number of primitives

$$l^{k+1} = \underset{l \in L \setminus \hat{L}^k}{\operatorname{argmax}} F(\hat{L}^k \cup \{l\}) - F(\hat{L}^k)$$





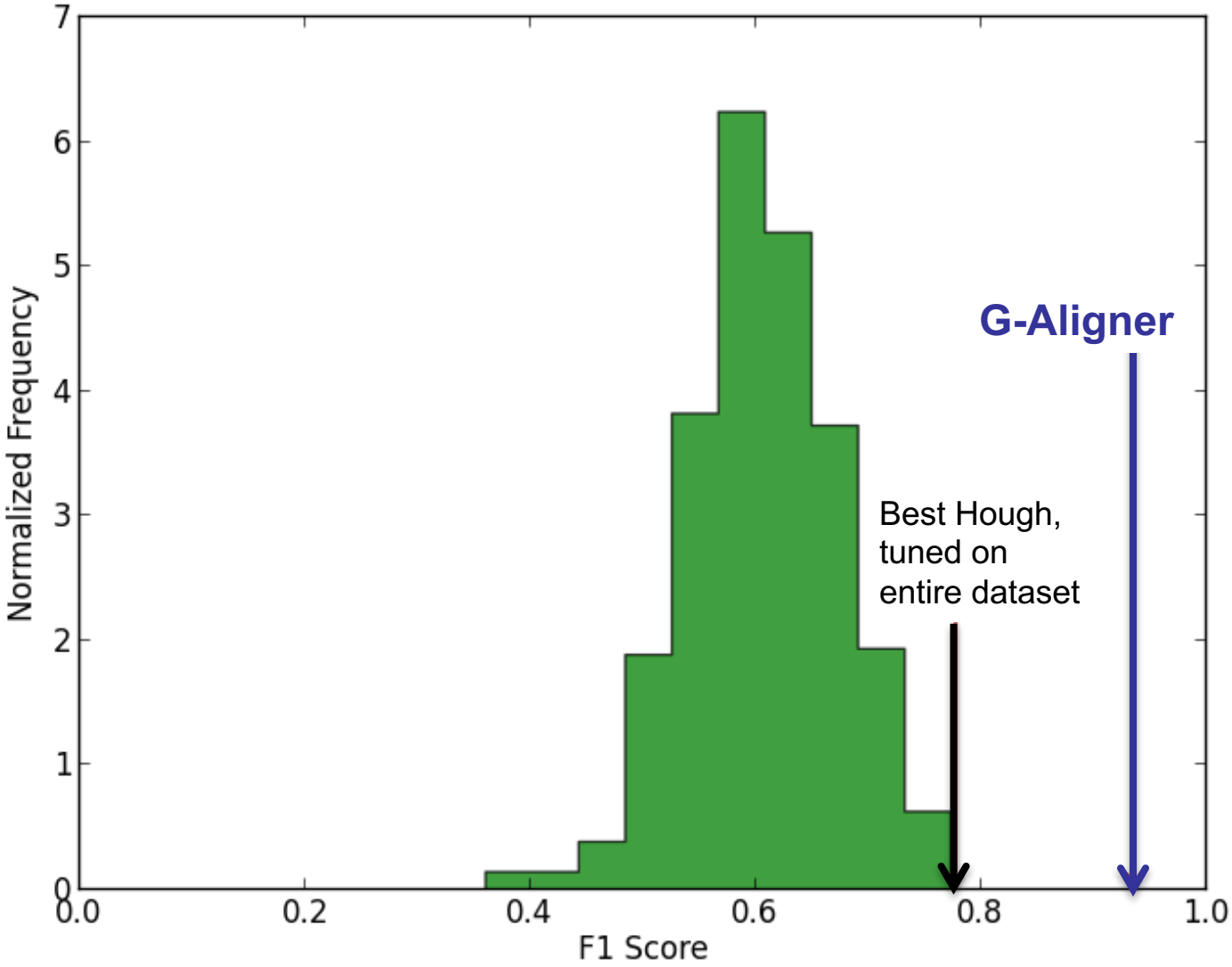
Experiments

Dataset (100 questions, 482 alignments)

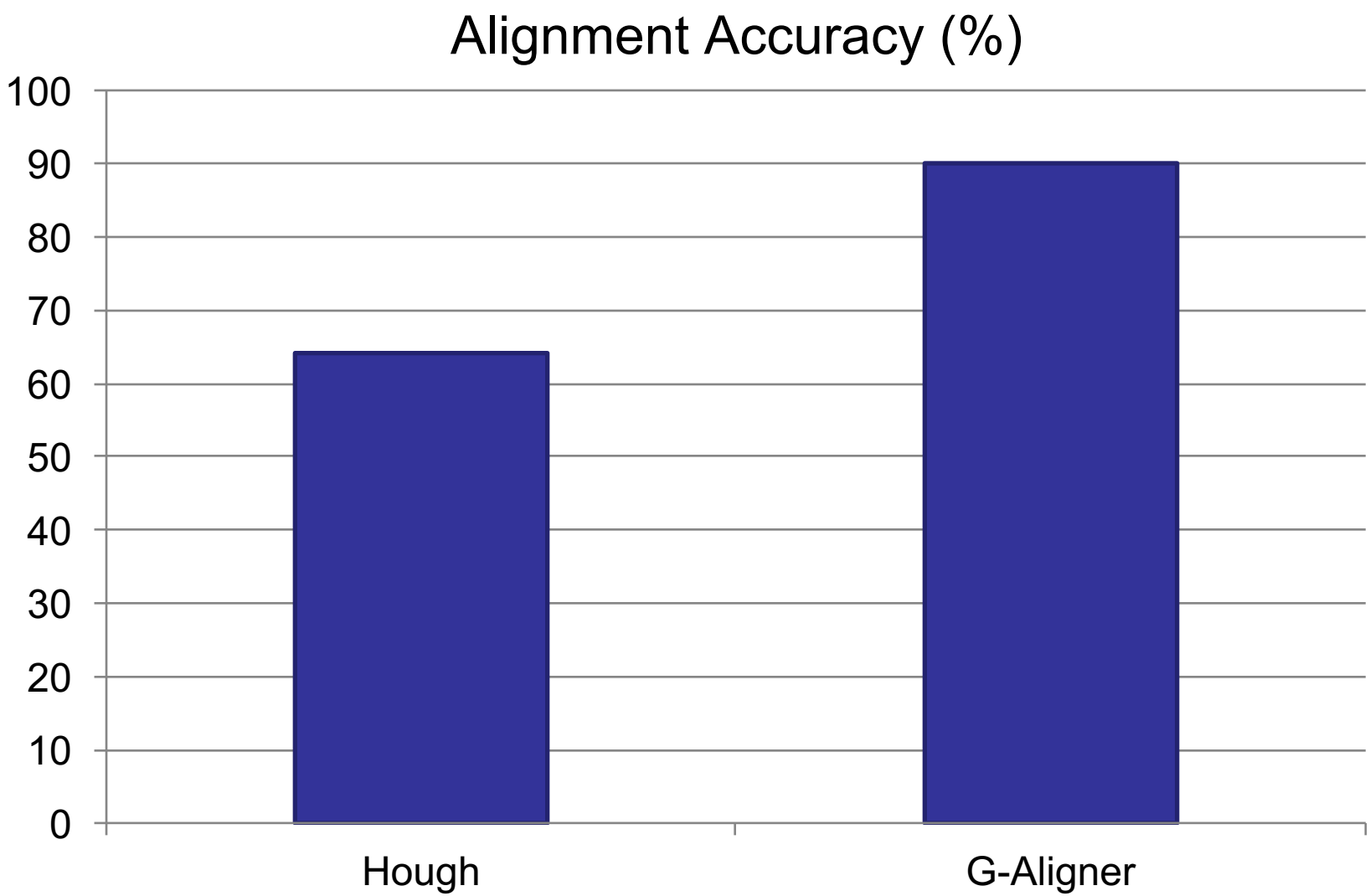
- Questions compiled from four websites for high school geometry (RegentsPrepCenter, EdHelper, SATMath, SATPractice)
- Manually recorded ground truth for visual primitives and textual alignment
- Dataset can be downloaded at:

cs.washington.edu/research/ai/geometry

Experiments: Detecting Primitives



Experiments: Alignment Accuracy



Demonstration

G-Aligner Demonstration

trusty.cs.washington.edu:9239/vpsel/

Google

G-Aligner

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Custom Problem

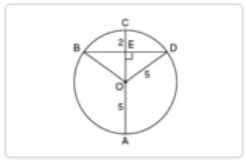
Upload your own problem by specifying image URL and question text.

Browse...

OR

Examples

Click an image below.



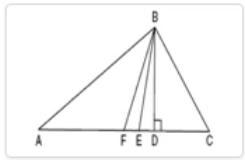


Diagram Understanding in Geometry Questions

To Appear in AAAI '14

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Conclusion & Future Work

- Diagram understanding
  - Overproduce lines and circles via Hough
  - Find a “good” subset
  - Objective function uses text and diagram

- Textual understanding
- Knowledge representation of both text and diagram
- Inference engine to solve questions

Thank you!

Diagram Understanding in Geometry Questions

For more information, please visit:

cs.washington.edu/research/ai/geometry

Hough Transform